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Single versus multi-domain analysis in diffusion equation modeling for sound field analysis of distinct room shapes

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ABSTRACT

Diffusion equation modelling (DEM) has gained popularity recently in the application of room acoustics analysis mostly due to its computational efficiency. The method has been used in proportionate rooms confidently, while some research has highlighted its applicability in disproportionate room typologies with certain assumptions. In this study specifically, coupled rooms and long enclosures that cannot be very well defined by statistical theory, or under-estimated by ray tracing are studied by DEM analysis. The diffusion coefficient, dependent upon the volume and surface absorption area of individual domains, is one major indicator in this analysis. Defining domains in coupled volumes or in multiple long spaces coupled each other is an important problem. The limits where the domains can be considered as a single space or as a multi-domain system are quite vague and necessitate a systematic investigation. Some real structures including multi-volume monuments and two-tracked subway stations are examined in that respect, by applying DEM in a finite element medium. The results of this investigation are assistive in regards to the coupling factor limits when different volumetric and aperture size relations are established. The pre-knowledge of applying either a single domain or a multi-domain solution in a specific combination of volumes will augment the speed of analysis.

Keywords: Diffusion equation model, room acoustics simulations, disproportionate rooms

1. INTRODUCTION

Sound energy decay analysis is essential in room acoustics predictions for predicting key characteristics of an enclosure. Alternative modeling and analysis methods are developed initially to estimate acoustical indicators for proportionate rooms with diffuse sound field. However, in the case of disproportionate room ratios or when multiple sub-volumes are coupled to each other, the interior sound field is much more complex and statistical theory is not applicable. Such distinct room shapes and configurations including coupled spaces and long enclosures have been investigated for several decades and a number of theoretical formulae have been established. Among those diffusion equation model has found to be a reliable (1-3) and practical method which is further discussed within this study.

One advantage of diffusion equation model is that, meshing size takes its value from the mean free path (MFP), rather than the wavelength, thus reducing the size of the model and thereby increasing the computational speed. While, diffusion coefficient takes into account the room morphology via its mean free path (4). In that sense diffusion coefficient becomes an important variable within a DEM solution. The critical issue is that when it comes to coupled volumes the decision on defining one or multiple diffusion coefficients may affect the results, which depends on the strength of coupling in between different spaces through apertures. In order to understand the conditions, when the coupled volumes behave as distinct spaces or a single volume, and that multiple diffusion coefficients should be defined, different cases are investigated in this research. Among those cases multi-domed

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superstructures and a two-track subway station are discussed over their single and multi-domain DEM solutions.

2. METHODOLOGY

This study applies one of the most recent methods in room acoustics that is the diffusion approximation. Diffusion equation model (DEM) can be solved analytically for simple geometries, and numerically for complex enclosures. DEM is based on the propagation of sound particles with the same constant energy, propagating along straight lines and striking walls or scattering objects. In this section, the governing interior and boundary equations applied to case structures are presented.

2.1 Interior Diffusion Equation

In a room region or domain (V) with time-dependent energy density (w) at a position (r) and as a function of time (t) the sound energy flow vector (J) caused by the gradient of the sound energy density can be expressed by Fick's law;

$$J(r,t) = -D\nabla w(r,t) \tag{1}$$

where D is the diffusion coefficient, which takes into account the room morphology via its mean free path (λ) given by (4);

$$D = \frac{\lambda c}{3} = \frac{4Vc}{3S} \tag{2}$$

where V is the volume of the room, S is the total interior surface area of the room and c is speed of sound. In the presence of an omni-directional sound source *q(r,t)* within a domain (V), the sound energy density w changes per unit time as;

$$\frac{\partial w(r,t)}{\partial t} - D\nabla^2 w(r,t) = q(r,t)\,, \quad \in V \tag{3}$$

Lastly, in a time dependent solution of DEM the energy flow levels (5) can be obtained by;

$$J_L(r,t) = 10\log_{10}\left\{\left[\frac{\partial w(r,t)}{\partial x}\right]^2 + \left[\frac{\partial w(r,t)}{\partial y}\right]^2 + \left[\frac{\partial w(r,t)}{\partial z}\right]^2\right\}^{1/2} \tag{4}$$

2.2 Boundary Conditions

The effects of enclosing room surfaces can be analytically expressed by boundary equations defined on the boundary surfaces (*S*). The boundary condition established to include energy exchanges on enclosing surfaces is (4-6);

$$J(r,t)\cdot n = -D\nabla w(r,t)\cdot n = A_X c w(r,t)\text{, on } S \tag{5}$$

where A_x is an exchange coefficient, or the modified absorption factor, expressed as follows (34);

$$A_X = \frac{\alpha}{4(1-\alpha/2)} \tag{6}$$

where α is the absorption coefficient of the specific surface or boundary. DEM with modified absorption factor (6) is capable of modeling rooms with low absorption, as well as for mixed boundary conditions associated with high absorption for specific room surfaces. This study utilizes Eq. (5) in Eq. (6) and the resulting system boundary equation is as follows;

$$-D\frac{\partial w(r,t)}{\partial n} = \frac{c\alpha}{4(1-\alpha/2)}w(r,t), \text{ on } S \tag{7}$$

For application of DEM in a multi-domain solution, there exists another boundary condition applied for coupling apertures that is the continuous boundary condition (7). The aperture boundary surface at intersection of two domains can be defined by the following;

$$\hat{n} \cdot \left[D_1 \nabla w(r_b, t) - D_2 \nabla w(r_b, t) \right] = 0 \tag{8}$$

which represents a continuity boundary condition on interior boundaries at the aperture position, where D_1 is the diffusion coefficient in the primary room, and D_2 is the diffusion coefficient for the secondary room.

3. MATERIALS

3.1 Multi-domed Superstructures

Multi-domed monuments are interesting venues for acoustical studies as they have many subspaces that are coupled to each other through arches, augmenting the potential for non-exponential energy decay formation. Two sacred spaces, namely Süleymaniye Mosque and Hagia Sophia in İstanbul, are selected in that respect in search of application of proper diffusion coefficients within DEM. In order to implement the DEM numerically in a finite element medium, initially the acoustical models of each structure are built. The effect of coupling of different sub-volumes are searched in single and multi-domain solutions. Thus, first, results are obtained for the single domain, meaning a single diffusion coefficient, Eq. (2), assigned for the whole structure. Second, specific diffusion coefficients in relation to their mean free paths (MFP) are defined for sub-volumes or sub-domains. Coupling apertures of both structures are the arches that connect sub-spaces to each other, sheltered mostly with domes or vaults of different sizes. Meshed models are fine-tuned with field test results taking the reverberation time into account (3,8).

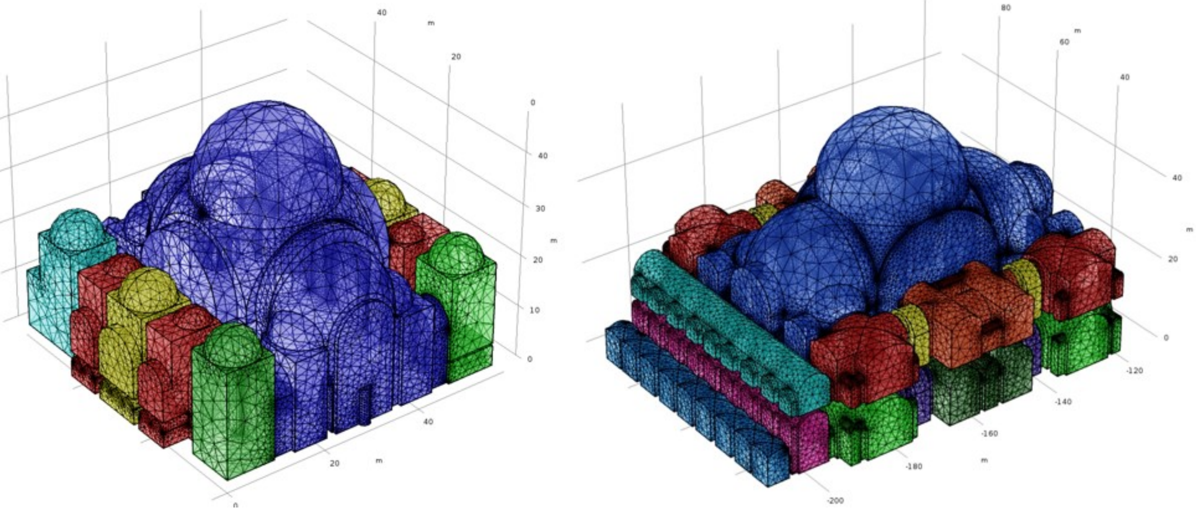


Figure 1 - Süleymaniye Mosque mesh model (on the left); Hagia Sophia mesh model (on the right); different domains indicated with individual colors

The geometric model of Süleymaniye Mosque has 164,468 linear Lagrange-type mesh elements, while Hagia Sophia model has 691,865 linear Lagrange-type mesh elements (Fig. 1). Minimum mesh size is 0.62 m for Süleymaniye mosque and 0.39 m for Hagia Sophia. Maximum mesh size is 4.96 m for Süleymaniye Mosque and 5.31 m for Hagia Sophia. As long as the maximum mesh size is smaller than the MFP of the room, the DEM is applicable. In this case the range of mesh sizes are in between 1/4 to 1/11 of MFP for Süleymaniye Mosque and 1/4 to 1/14 of MFP for Hagia Sophia. Thus, maximum mesh sizes of both models satisfy the MFP criteria for the DEM. The time-dependent simulation takes approximately 13 mins on a computer with Intel(R) Xeon(R) E5-1650 CPU, @ 3,60 GHz processor for Süleymaniye Mosque and 1h/4min for Hagia Sophia models. Table I lists the volume and the total surface area of individual domains. Accordingly, the mean free paths (MFP) and diffusion coefficients (D) are calculated for different domains of case structures (3).

Table I. Volume (V), surface area (S), mean free path (MFP) and diffusion coefficient (D) information for single and multi-domain scenarios of Süleymaniye Mosque and Hagia Sophia models

Süleymaniye	V (m^3)	S (m^2)	MFP (m)	D	Hagia Sophia	V (m^3)	S (m^2)	MFP (m)	D
Single Dom.	73,848	18,293	16.2	1,846	Single Dom.	145,020	38,579	15.0	1,720
D0	55,525	10,873	20.4	2,336	D0	95,960	17,647	21.8	2,487
D1	2,240	1,300	6.9	788	D1	2,575	1,241	8.3	949
D2	2,279	1,115	8.2	934	D2	625	468	5.3	611
D3	1,892	1,129	6.7	766	D3	4,430	2,158	8.2	939
D4	3,063	1,557	7.9	900	D4	6,771	3,434	7.9	902
D5	2,820	1,289	8.8	1,000	D5	2,395	1,728	5.6	635
					D6	4,254	2,328	7.3	836
					D7	2,499	1,190	8.4	960
					D8	782	584	5.4	613
					D9	3,625	1,938	7.5	855

3.2 Subway Stations

Subway stations are long enclosures with different acoustical properties from normal rooms, due to their disproportionate room ratios. The specific acoustic properties in long enclosures have been investigated for several decades and a number of theoretical formulae have been established (9-11). Sound attenuation in real subways is even more complicated. In search of reliable methods of sound field estimations in long and disproportionate spaces, this study applies DEM on two case subway stations from Üsküdar-Ümraniye metro line in İstanbul.

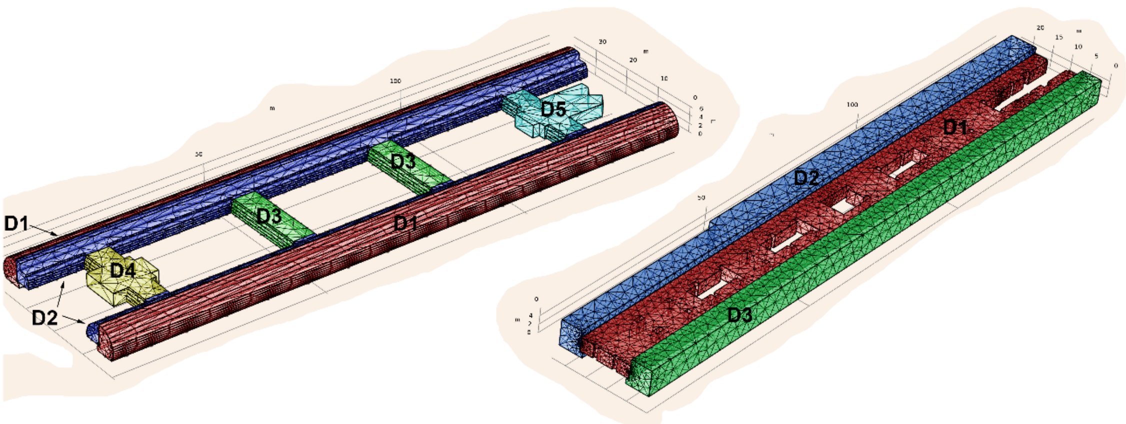


Figure 2 – Station BAG mesh model (below); station USK mesh model (above); different domains indicated with individual colors

Out of two stations, station BAG has a rectangular cross section, packs system, two tracks and connecting corridors, and station USK has a circular cross section and two tracks with a connecting hall. For computation of DEM in a finite element medium the meshed models of both structures are generated. The meshed model of station BAG has total of 282,701 linear, Lagrange-type mesh elements with maximum mesh size of 4.1 m. While meshed model of station USK has total of 94,046 linear, Lagrange-type mesh elements with maximum mesh size of 4.9 m (Fig. 2). In this case the maximum mesh sizes of both BAG and USK are smaller than their estimated MFPs, so the DEM is applicable (Table II).

Table II. Volume (V), surface area (S), mean free path (MFP) and diffusion coefficient (D) information for single and multi-domain scenarios of (D1-D5) of station BAG and station USK

BAG	V (m3)	S (m2)	MFP (m)	D	USK	V (m3)	S (m2)	MFP (m)	D
Single Domain	12,988	11,143	4.7	533	Single Domain	14,187	11,250	5.0	577
D1	3,575	2,995	4.8	545	D1	4,274	4,674	3.7	418
D2	2,084	2,810	3.0	340	D2	5,419	3,875	5.6	640
D3	368	393	3.8	429	D3	4,459	3,548	5.0	575
D4	504	521	3.9	442					
D5	691	694	4.0	455					

4. RESULTS

4.1 Sound Energy Flow Decays and Coupling Factor

In a previous study the occurrence of multi-slope decay formation in multi-domed superstructures are presented (7). It is known that energy flow decays figuring energy flow dips are correlated with the turning points in a non-exponential energy decay (5). The effects of architectural variables including, coupling apertures, volume and geometry of domains can better be analyzed through energy flow distribution in a DEM solution. In the modeling and computation phase, it is critical to correctly define the individual domains with different diffusion coefficients for reliability of DEM solution.

In Fig. 3 single versus multi-domain DEM energy flow decays of Süleymaniye Mosque and Hagia Sophia are presented for sample source-receiver configurations, where double-slope energy decays are previously detected through Bayesian decay parameter estimations (7). The convex forms of energy flow decays indicate an energy return, either a full return or partial. Deeper the energy flow dip, the energy flow pattern approaches to a complete return (3), indicating a higher difference between decay levels and decay times of different slopes. A sample source-receiver position is S_1R_4 for Hagia Sophia (Fig. 3), where the difference between energy flow dips in a single domain solution versus multi-domain solution of DEM is greater.

In an earlier study Billon et al. (12) applied mean coupling factor (k) for quantifying the degree of acoustical coupling. Mean coupling factor is defined as follows;

$$\kappa = \sqrt{\frac{S_c^2}{(S_c + A_R)(S_c + A_S)}} \tag{9}$$

where S_c is the coupling area, A_R is the equivalent absorption area of the receiving room and A_S is the absorption area of the source room. According to that k ≈ 1 denotes a strong coupling, while k ≈ 0 indicates a weak coupling. A strong coupling means that the aperture sizes are big enough, so the two volumes should not be considered as separate domains, but instead should be considered as a single volume. Conversely, a weak coupling indicates that the apertures are small enough to consider the two volumes as part of a multi-domain system.

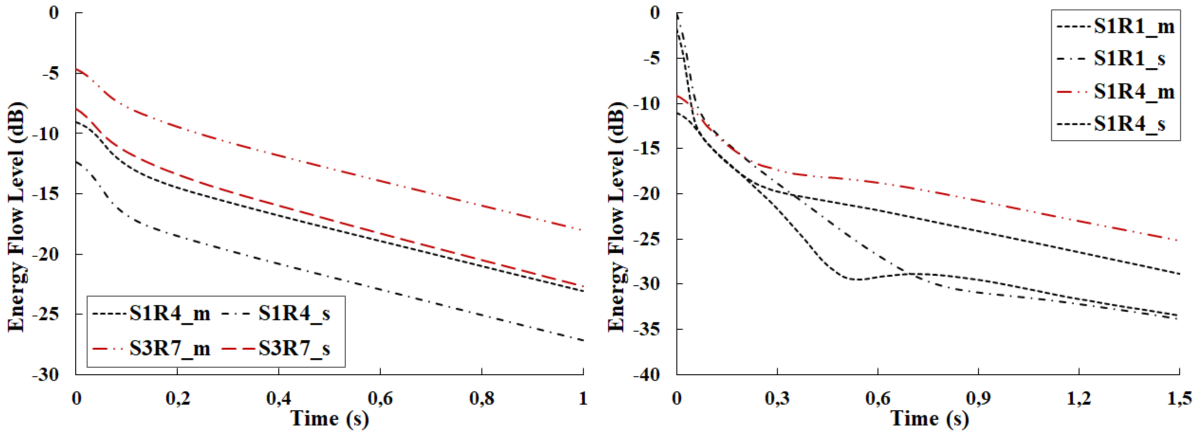


Figure 3 - DEM solutions for single ('s') versus multi-domain ('m') energy flow decay results; for 1 kHz, computed at different source (S) and receiver (R) configurations, Süleymaniye Mosque (on the left), Hagia Sophia (on the right)

A weak coupling necessitates the assignment of individual diffusion coefficients to each volume as in the case of Süleymaniye Mosque and Hagia Sophia. Table III lists k values obtained by Eq. (9) for specific source-receiver configurations tested in case structures. As can be observed in Table III, considering sub-volume to main volume coupling (Table I and Table II), the coupling factors are smaller than 0.30 for Süleymaniye Mosque and much smaller than 0.10 for Hagia Sophia. Although the limits of weak to strong coupling are not well defined, in a strong coupling indication by mean coupling factor of 1, the values lower than 0.30 can securely be assumed to be a weak coupling. Thus, the individual volumes should be treated as individual domains with specific diffusion coefficients in DEM simulation of both cases. There is still a considerable difference in the mean coupling factors between Süleymaniye Mosque and Hagia Sophia. This is mainly due to the larger coupling apertures (arches) of Süleymaniye Mosque that divides main volume from smaller sub-volumes, in comparison to those of Hagia Sophia. For that reason, the energy flow dips or flow returns of Süleymaniye Mosque, are not as sharp as those of Hagia Sophia (Fig.3).

Table III. Mean coupling factors (*k*) for specific source (S) receiver (R) configurations in relation to coupling aperture area and absorption areas of individual domains (D) coupled to each other by arches for Süleymaniye Mosque and Hagia Sophia

Süleymaniye (*k*)	S_1R_4	S_3R_7	S_4R_5	S_2R_1	Hagia Sophia (*k*)	S_1R_4	S_1R_5	S_2R_1	S_2R_2
D0-D1	0,25	-	-	-	D0-D1	0,05	-	-	-
D0-D2	-	0,27	-	-	D0-D3	-	0,07	-	-
D0-D4	-	-	0,28	-	D1-D0	-	-	0,05	0,05
D2-D0	-	-	-	0,27					

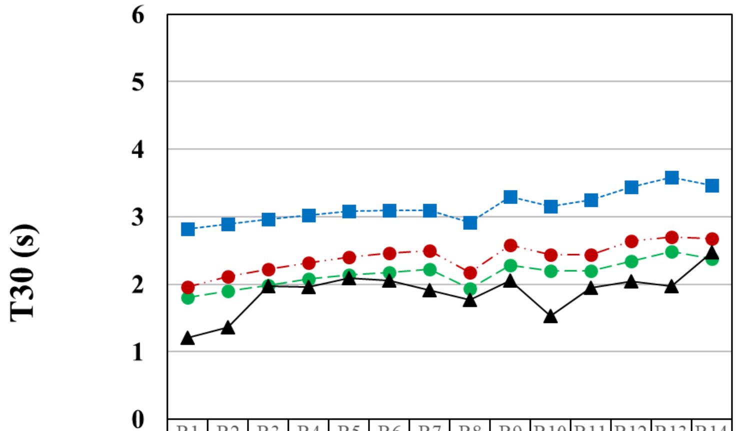
4.2 Decay Rate Estimates in Long Enclosures

In this section single and multi-domain DEM solutions of case subway stations are comparatively evaluated over their T30 results for different receiver positions. The reason is basically for understanding if coupling of volumes, in form of tunnels to sub tunnels and tracks to halls have significant effect on the outcomes of different DEM solutions. The model of station BAG is tuned by its field test results and known sound absorption coefficient data of applied materials within the station. On the contrary to their geometrical features, interior finish materials are identical in both station BAG and station USK that are ceramic tiles and painted concrete on walls and track tunnels, stone tile flooring and mineral wool backed perforated metal ceilings (13).

Fig 8. compares field test, single and multi-domain DEM and ray-tracing results of T30 in station

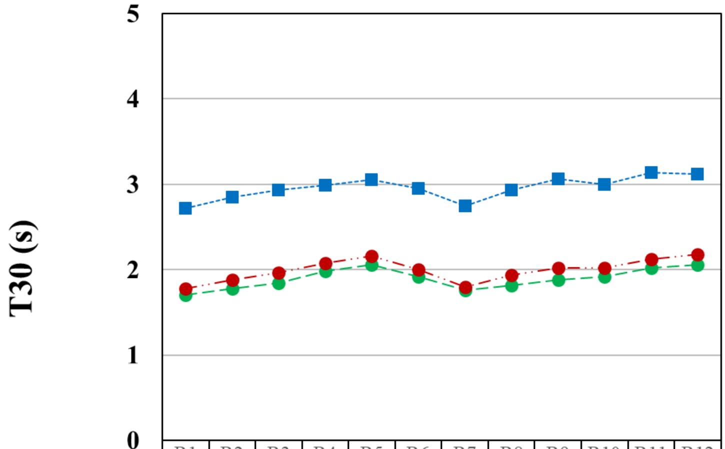
BAG for different receiver positions. For both field test results and simulated decay rates the reverberation increases as the distance from the source increase. According to that decay rates of multi-domain DEM solution are 0,20 s to 0,40 s higher than single domain DEM solution. On the other hand, single domain DEM results are much closer to field test results when they are compared to multi-domain DEM results and ray-tracing. This means the station BAG space does not act as a coupled-volume system and the apertures connecting different volumes (halls and two tunnels) to each other are not small enough to cause non-exponential sound energy decay as previously observed in different structures. Ray-tracing over-estimates the decay rates and there are abrupt jumps in reverberation times on different receiver positions. In overall, single domain DEM results among others, are much reliable and highly correlated with field test results.

Fig. 5 compares single and multi-domain DEM and ray-tracing results of T30 in station USK for different receiver positions. Multi domain DEM decay rate results in overall parameters are 0.1 s to 0.2 s higher than single-domain DEM solution. This slight variation is even smaller than the deviations observed in station BAG. This is due to the fact that USK station is even much less fragmented than station BAG (Fig. 2) and the station space together with its two track volumes and central platform area behaves as a single volume. Thus, multi-domain investigation is proved not to be necessary.



	R1	R2	R3	R4	R5	R6	R7	R8	R9	R10	R11	R12	R13	R14
BAG_DEM_s	1,80	1,90	1,98	2,08	2,14	2,18	2,22	1,94	2,28	2,20	2,20	2,34	2,48	2,38
BAG_DEM_m	1,96	2,12	2,22	2,32	2,40	2,46	2,50	2,18	2,58	2,44	2,44	2,64	2,70	2,68
BAG_RAY TRACING	2,82	2,89	2,96	3,02	3,08	3,10	3,10	2,92	3,30	3,16	3,25	3,44	3,59	3,47
BAG_MEAS	1,20	1,37	1,97	1,95	2,10	2,06	1,91	1,77	2,06	1,53	1,95	2,05	1,97	2,47

Figure 4 - Comparison of field test, single and multi-domain DEM and ray-tracing results of T30 in station BAG for different receiver (R1 to R14) positions for 1 kHz



	R1	R2	R3	R4	R5	R6	R7	R8	R9	R10	R11	R12
USK_DEM_s	1,70	1,78	1,84	1,98	2,06	1,92	1,76	1,82	1,88	1,92	2,02	2,06
USK_DEM_m	1,78	1,88	1,96	2,08	2,16	2,00	1,80	1,94	2,02	2,02	2,12	2,18
USK_RAY TRACING	2,72	2,85	2,93	2,99	3,05	2,95	2,75	2,93	3,06	3,00	3,14	3,12

Figure 5 - Comparison of single and multi-domain DEM, and ray-tracing results of T30 in station USK for different receiver (R1 to R12) positions for 1 kHz

5. CONCLUSION

In this study diffusion equation model is discussed over rooms with particular room shapes including coupled spaces and disproportionate long rooms. Main concern is the comparison of single-domain to multi-domain DEM solutions in complex structures, for predicting the reliable modeling technique in different cases. Results of multi-domed superstructures highlight that the multi-domain DEM analysis results are much indicative in multi-rate decay analysis in comparison to the single-domain solutions. In single domain solutions, at some particular positions the energy dips are not observed, though the same locations indicate an obvious dip over their multi-domain energy flow decays. There is always a difference between single versus multi-domain DEM solutions. As the coupling factor gets weaker, for instance by smaller apertures, the difference is much obvious. Thus, for both case structures with coupling factors below 0,30 the reliable method is found to be the multi-domain solution.

The optimization of materials in acoustical design of subway stations starts with the decay rate estimations and in that sense diffusion equation model is a practical method. In case subway stations single versus multi-domain DEM solutions are compared to field test and ray-tracing results of T30, in order to understand the effect of coupling and for defining the proper diffusion coefficients. Single-domain DEM results are found much compatible to field test results considering decay rates. The results highlight that for both case stations, the coupling of volumes in form of tunnels to inner corridors or central halls is not weak enough, meaning the aperture sizes are big and the equivalent absorption area is small. Thus, single domain DEM solution is found to be suitable for the case subway stations.

REFERENCES

1. Sü Gül Z, Xiang N, Çalışkan M. Investigations on sound energy decays and flows in a monumental mosque, J. Acoust. Soc. Am. 2016;140(1):344-355.
2. Sü Gül Z, Xiang N, Çalışkan M. Diffusion equation based finite element modeling of a monumental worship space. J. Comput. Acoust. 2017;25(4):1-16.
3. Sü Gül Z, Odabaş E, Xiang N, Çalışkan M. Diffusion equation modeling for sound energy flow analysis in multi domain structures. J. Acoust. Soc. Am. 2019;145(4):2703-2717.
4. Valeau V, Picaut J, Hodgson M. On the use of a diffusion equation for room-acoustic prediction. J. Acoust. Soc. Am. 2006;119(3):1504-1513.
5. Xiang N, Jing Y, Bockman AC. Investigation of acoustically coupled enclosures using a diffusion-equation model. J. Acoust. Soc. Am. 2009;126(3):1189-1198.
6. Jing Y, Xiang N. On boundary conditions for the diffusion equation in room acoustic prediction: theory, simulations, and experiments. J. Acoust. Soc. Am. 2008;123(1):145-153.
7. Xiang N, Escolano J, Navarro JM, Jing Y, Investigation on the effect of aperture sizes and receiver positions in coupled rooms. J. Acoust. Soc. Am. 2013;133(6):3975-3985.
8. Sü Gül Z, Çalışkan M, Tavukçuoğlu A, Xiang N. Assessment of acoustical indicators in multi-domed historic structures by non-exponential energy decay analysis. Acoust. Australia 2018; 46:181-192.
9. Kang J. Sound attenuation in long enclosures. Build. Environ. 1996;31(3):245-253.
10. Yang L, Shield BM. Development of a ray tracing computer model for the prediction of the sound field in long enclosures, J. Sound Vib. 2000;229 (1):133-146.
11. Li KM, Lu KK. 2004. Propagation of sound in long enclosures. J. Acoust. Soc. Am. 2004;116(5):2759-2770.
12. Billon A, Valeau V, Sakout A, Picaut J. On the use of a diffusion model for acoustically coupled rooms. J. Acoust. Soc. Am. 2006;120(4):2043-2054.
13. Sü Gül Z, Odabaş E, Çalışkan M. Acoustical assessment of subway stations by diffusion equation modelling. Proc 13th WESPAC; 11-15 November 2018; New Delhi, India 2018.