Investigations on multi-slope sound energy decays in domed structures

Zuhre Su Gul[a)]
Middle East Technical University, Department of Architecture, Ankara, 06800, Turkey; MEZZO Studyo, Ankara, 06800, Turkey

Ning Xiang[b)]
Graduate Program in Architectural Acoustics, School of Architecture, Rensselaer Polytechnic Institute, Troy, New York 12180
Mehmet Caliskan[c)]
Middle East Technical University, Department of Mechanical Engineering, Ankara, 06800, Turkey

Sound energy decays of coupled volume systems have attracted acousticians for many years. The interesting sound phenomenon is also applied in real cases as concert halls for influencing perception metrics to desired ranges. Theoretical studies of energy exchanges between coupled volumes with help of Bayesian inference aim to estimate parameters associated with multiple slope decays and characterize energy decays of multiple decay process. This study extends the definition of coupled volume system known to be two or more volumes with well defined enclosure limits coupled to each other with a coupling aperture. As specific combinations of different architectural volumes are highly correlated with acoustical coupling, domed structures are found to have a potential for multi slope investigations. Instead of multiple volumes with defined enclosures a single volume with specific geometric attributes is considered to be base for multiple-slope formation. Within this study a room-acoustic model of a multiple dome superstructure is established for multiple-slope decay analysis. Initial results indicate that double, triple or even more slope natures can be identified within the space at specific frequency bands.

[a)] email: zuhre@mezzostudyo.com
[b)] email: xiangn@rpi.edu
[c)] email: caliskan@metu.edu.tr

1 INTRODUCTION

Coupled volume systems have far been investigated for theoretical understanding of peculiar sound energy decay characteristics within such enclosures in order to better orient their use in real cases ranging from large concert halls to much smaller studios. Non-exponential energy decay regarding to its inherent properties of early and late decay/s in very early studies have found to be detrimental due to creating an unfamiliar acoustical impression then has become a design tool in providing a lingering reverberance while maintaining sense of intimacy and clarity.

Until very recently, the main focus of the studies has been the so-called double decay phenomenon which is an observable quality in previous studies in compare to multiple decays. Models and methods are developed with an aim of identifying double-slope decays and quantifying the degree of coupling. The proposed models adapt either one or couple of principle approaches which are statistical theory[1-2], statistical energy analysis[3], diffusion models[4-5], wave theory[6] and geometrical acoustics[7-8]. The state of theory and manual computation and analysis tools have restricted much work to identify only two decay slopes while there is still the probability of the existence of multiple-slope decays within investigated sound energy decays. In a recent study Xiang et al.[9] take a step forward and investigate multiple-slope decays in sound energy decays of coupled-volume systems. Energy exchanges between coupled-volume systems with help of Bayesian inference aim to estimate parameters associated with multiple-slope decays and characterize energy decays of multiple decay processes. In that effort an approach termed Bayesian information criterion (BIC) is introduced to accomplish the decay model selection from a set of competing models.

The non-exponential energy decay is generally expected from a coupled-volume system which is known to be two or more volumes with well defined enclosure limits coupled to each other with a coupling aperture. The major issue in this study is that even single-volume systems with specific architectural compositions can also form non-exponential energy decays. As specific combinations of different architectural volumes are highly correlated with acoustical coupling, domed structures are found to have a potential for featuring multi-slope decays. Instead of multiple volumes with defined enclosures a single volume with specific geometric attributes is considered to be base for the multiple-slope formation. Being part of a larger research project, in this study a room-acoustic model of a multiple dome superstructure is established for multiple-slope decay analysis. Bayesian parameter estimation is applied in search of multiple-slope sound energy decay within this single-volume system and results are discussed.

2 CASE STUDY

2.1 Süleymaniye Mosque, İstanbul

In the extent of a broader research project acoustical fields within couple of multiple dome superstructures are to be investigated by computer model technique and field measurements. This paper presents assessment of multiple decay formation within Süleymaniye Mosque. In this initial phase the room impulse responses are obtained using a ray tracing model. Future work will include field measurements.

Süleymaniye Mosque complex, built in 1559 in İstanbul, is the largest of the Ottoman building enterprises of the time. The domed main structure, together with supplementary spaces sheltered with smaller sized domes is product of a modular system of abstract forms (Fig. 2). The inner plan of the mosque is a rectangle measuring 63 by 69 m. The mosque is covered centrally by a single dome which is supported on two sides by semi domes. Main dome rests on a circular wheel with a diameter of 26 m. The height from the foundation to the impost is 33.70 m. The inner rise of the dome is 14.05 m, and thus the height of the dome from the ground to the keystone is 47.75 m. The interior walls are faced with stone revetments. The ceilings of the pulpits and the royal box, the domical superstructure, and the pendentives are painted. Painted brick domes are decorated with gold foiled pen carvings (Fig. 1). The mihrab and the minbar (pulpit) are of carved white marble and there are stained glass windows on the sides. Wood in interior is mostly used for doors, window frames and furniture. Floor finish of the mosque is carpet with straw backing as stated. Within the Mosque muqarnas/stalactites are used as transition elements as skirting of half domes to enhance the diffusion in mostly curvilinear and concave transition planes by fragmenting the surfaces into much smaller pieces.

2.2 Ray Tracing Model

A graphical model of the Mosque is generated in SketchUp for use with ODEON room acoustics software version 11.21. The simplified model made up of 3-D face elements is comprised of 38,478 plane surfaces. In the process of modification of the geometry the graphical model is preserved in great detail in order to maximize the effects of domes and scattering surfaces, while keeping the limits of minimum surface dimensions dictated by the software (Fig. 2). Estimated acoustical volume of the mosque is 134,342 m³.

The materials that are applied to room surfaces are listed in Table 1 with corresponding sound absorption coefficient data. Scattering coefficients ranging in between 0.1 to 0.5 are attained to surfaces according to differing surface irregularities. In calculation parameters number of late rays is defined to be 329,283 and the impulse response length is set to 15000 ms. Ray tracing is used majorly in sound path analysis. One omni-directional source is defined in front of mihrab at 1.50 m depicting standing imam (Fig. 3). Twenty two receivers are distributed throughout the prayer zone as of standing prayer positions. An impulse response is obtained at each receiver position, which is then exported from ODEON for computational analysis as discussed in following section.

3 COMPUTATIONAL ANALYSIS

3.1 Decay Parameter Estimation

Decay rate and decay order estimation of multiple slopes has always been a challenging task. Methods for identifying double-slope decays and the degree of coupling are proposed. Visual inspection, comparisons of linear-fits of different portions of logarithmic decay functions or applying ratio based quantifiers without considering their absolute values are proved to be scientifically ineffective by some recent studies especially when the decay profile has more than two slopes[10].

Bayesian analysis has long been applied by Xiang et. al[11-13] and reliable methods in characterizing sound energy decays consisting of one or two slopes has been presented. In a recent study[9] authors adopted Bayesian formulation in characterization of multiple-slope decays. Two Bayesian formulations are compared using the room impulse responses obtained from coupled scale models. The use of Bayesian Information Criterion (BIC) is found to be an efficient approach in solving Bayesian model selection problems that are suitable for estimating the number of exponential decays. Parametric model describing Schroeder decay function contains decay parameters of A_s and T_s, where A_s is the linear amplitude parameter and related to the level of individual exponential decay terms, T_s is the decay time associated with the logarithmic decay slope of individual exponential decay terms, with s = 1, 2,…, S, and S is the maximum number of exponential decay terms, also termed the decay order.

Evaluating degrees of the curve fitting often leads to over-parameterized models, since increased decay orders always improve curve fitting. The scientifically thorough solution is to evaluate the Bayesian evidence which automatically encapsulates the principle of parsimony, and quantitatively implements Ockham's razor, which prefers simpler models and penalizes over-fitting. In the scope of the energy decay analysis among a set of decay models the model yielding the largest BIC value is considered to be the most concise model providing the best fit to the decay function data and at the same time capturing the important exponentially decaying features evident in the data. In following section multiple decay rates are searched by applying BIC for ranking the competing decay models, such as double-slope, triple-slope or quadruple-slope decay models.

3.2 Results

Ray tracing model including source and receivers is initially simulated in ODEON software. Global estimate reverberation time (T30) for the entire space and multi point responses for each receiver are obtained. Global T30 results are very much in line with T30 averaged for 22 receiver points. Average values of T30 over octave bands together with maximum and minimum values over the receiver zone are depicted in Figure 4.

Room impulse responses obtained from ODEON simulation of the modeled Mosque are then computed within the framework of Bayesian approach. The decay order is determined by comparing BIC values of each model with highest score and best fit of model with the Schroeder decay function of real data. In Table 2 in accordance with estimated decay number, T_s over octave bands impulse for five representative receiver points. One case is selected from receiver positions with multiple decay formation for illustration of decay parameters. A_s, T_s and BIC values of quadruple slope decay model for collected impulse at Receiver no. 10 (R_{10}) are given in Table 3. Comparison between Schroeder decay curve and the model curve using quadruple-slope decay model along with decomposed decay lines and turning points are presented in Figure 5.

4 DISCUSSION

In the study of multiple decay phenomena a mega Mosque with multiple domes is computer-simulated. According to that T30 average for low frequencies (125-250 Hz) is 10.51 s, mid frequency average (500-1000 Hz) is 7.02 s and high frequency average (2000-4000 Hz) is 3.30 s. ODEON ray tracing model indicates that there is a bass build up within the space while reverberation gradually decreases towards the high end of the frequency spectrum. Simulated room impulse responses are used for multiple decay investigation. Double, triple and quadruple decays are observed above 2 kHz. The decay order selection is based on BIC comparison of models that fit well with the Schroeder decay curve. A sample group of calculated decay parameters for receiver locations are presented in Table 2 and 3. Simulated T30 values among different receiver locations show an even distribution. Estimated decay numbers together with T_s values per octave band obtained from Bayesian analysis are pretty much similar for twenty two receiver positions. T_1 values are close to the simulated T30 values in decays where multiple slopes are observed. $T_{2, 3...}$ are at least twice as high as T30 calculated in room acoustics software raising the question of whether this single number reverberation time estimations of ray tracing models are reliable or not considering specific cases.

In this case multiple slopes are not observed in low and mid frequency ranges. This may be due to the fact that the sound field is much diffuse in mid to low frequency range in compare to higher frequencies. The high frequency content of sound scattered from highly reflective wall and ceiling/dome surfaces are encountered with the absorption on the floor. As a result of the large area it covers, the carpet floor finish composes the highest Sabine at frequencies over 2kHz. The concentration of sound absorption at specific locations, in other words the uneven distribution of absorption coefficients, creates the non-diffuse field at corresponding ranges of the frequency spectrum. Multiple decay, or mostly double decay, phenomena has been searched for coupled spaces until recently. Even a single space with undiffused sound field can provide the circumstances for multiple-slope decay formation as in this case.

In this study at one source location, corresponding to imam in front of mihrab, impulse responses are obtained for twenty two receiver locations. A further study should include source at different locations, especially at central points below main and secondary domes, for observing different effects it creates in the sound energy decay. Being part of a larger research project with couple other mega structures to be analyzed, in this paper one real case of multi-domed systems is presented. Computer simulation technique is utilized for the initial stage of the case assessment. Real size field measurements will further support validating the findings and enable comparison between real and simulated data in multiple decay investigations.

5 REFERENCES

1. C. F. Eyring, "Reverberation time measurements in coupled rooms", *J. Acoust. Soc. Am.*, **3**(2), 181-206, (1931).

2. L. Cremer and H. A. Muller, "Coupled rooms", Chap. 11.3 in *Principles and Applications of Room Acoustics, v.1*, Elsevier, London, 261-292, (1978).

3. E. N. Wester and B. R. Mace, "A statistical analysis of acoustical energy flow in two coupled rectangular rooms", *Acta Acustica*, **84**, 114-121, (1998).

4. A. Billon, V. Valeau, A. Sakout and J. Picaut, "On the use of a diffusion model for acoustically coupled rooms", *J. Acoust. Soc. Am.*, **120**(4), 2043-2054, (2006).

5. N. Xiang, Y. Jing and A. C. Bockman, "Investigation of acoustically coupled enclosures using a diffusion-equation model", *J. Acoust. Soc. Am.*, **126**(3), 1189-1198, (2009).

6. C. M. Harris and H. Feshbach, "On the acoustics of coupled rooms", J. *Acoust. Soc. Am.*, **22**(5), 572-578, (1950).

7. J. S. Anderson and M. B. Anderson, "Acoustic coupling effects in St Paul's Cathedral, London", *J. Sound and Vib.*, **236**(2), 209-225, (2000).

8. L. Nijs, G. Jansens, G. Vermeir and M. Voorden, (2002) "Absorbing surfaces in ray-tracing programs for coupled spaces", *Applied Acoustics*, **63**, 611-626, (2002).

9. N. Xiang, P. M. Goggans, T. Jasaand P. Robinson, "Bayesian characterization of multiple-slope sound energy decays in coupled-volume systems", *J. Acoust. Soc. Am.*, 129(2), 741-752, (2011).

10. N. Xiang, P. Robinson and J. Botts, "Comments on 'Optimum absorption and aperture parameters for realistic coupled volume spaces determined from computational analysis and subjective testing results'" [in J. Acoust. Soc. Am. 127, 223-232 (2010)] (L), *J. Acoust. Soc. Am.*, **127**, 2539-2542, (2010).

11. N. Xiang and P. M. Goggans, "Evaluation of decay times in coupled spaces: Bayesian parameter estimation", *J. Acoust. Soc. Am.*, **110**, 1415-1424, (2001).

12. N. Xiang and P. M. Goggans, "Evaluation of decay times in coupled spaces: Bayesian decay model selection", *J. Acoust. Soc. Am.*, **113**, 2685-2697, (2003).

13. N. Xiang, P. M. Goggans, T. Jasa and M. Kleiner, "Evaluation of decay times in coupled spaces: Reliability analysis of Bayesian decay time estimation", *J. Acoust. Soc. Am.,* **117**, 3707-3715, (2005).

Table 1 - Sound absorption coefficients of materials applied within mosque interior surfaces

Material type	Octave band center frequency (Hz)					
	125	250	500	1000	2000	4000
Brick unglazed, painted	0.01	0.01	0.02	0.02	0.02	0.03
Marble	0.01	0.01	0.01	0.01	0.02	0.02
Red/grey stone	0.01	0.02	0.03	0.07	0.09	0.10
Doors, solid wood panels	0.10	0.07	0.05	0.04	0.04	0.04
Glazing	0.35	0.25	0.18	0.12	0.07	0.04
Carpet	0.12	0.17	0.23	0.41	0.59	0.61

Table 2 - T_s values over octave bands for estimated decay numbers; impulses obtained at R1, R5, R10, R15 and R20

Receiver number	Octave band center frequency (Hz)					
	125	250	500	1000	2000	4000
1	T_1 9.8	T_1 9.2	T_1 8.3	T_1 5.3	T_1 3.0	T_1 2.3
				T_2 7.8	T_2 5.1	T_2 4.7
					T_3 8.9	T_3 9.0
5	T_1 9.8	T_1 9.3	T_1 8.4	T_1 5.0	T_1 4.1	T_1 2.5
				T_2 8.0	T_2 5.7	T_2 4.0
					T_3 9.3	T_3 7.2
						T_4 13.3
10	T_1 10.0	T_1 9.4	T_1 8.5	T_1 5.5	T_1 3.9	T_1 2.3
				T_2 7.9	T_2 5.8	T_2 4.0
					T_3 9.6	T_3 8.1
						T_4 13.4
15	T_1 10.0	T_1 9.2	T_1 8.5	T_1 5.1	T_1 4.0	T_1 2.5
				T_2 8.2	T_2 5.8	T_2 4.0
					T_3 9.9	T_3 7.6
						T_4 14.7
20	T_1 9.9	T_1 9.2	T_1 8.2	T_1 5.5	T_1 3.3	T_1 2.5
				T_2 7.9	T_2 5.3	T_2 4.0
					T_3 8.9	T_3 7.0
						T_4 13.8

Table 3 - Decay parameters for impulse response collected at R_{10} filtered for 4kHz

Decay parameters	R_{10} 4kHz
A_0 (dB)	-107
A_1 (dB)	-6.2
T_1 (s)	2.3
A_2 (dB)	-9.0
T_2 (s)	4.0
A_3 (dB)	-24.1
T_3 (s)	8.1
A_4 (dB)	-37.8
T_4 (s)	13.4
BIC (Neper)	4702

Fig. 1 - Interior view from Süleymaniye Mosque
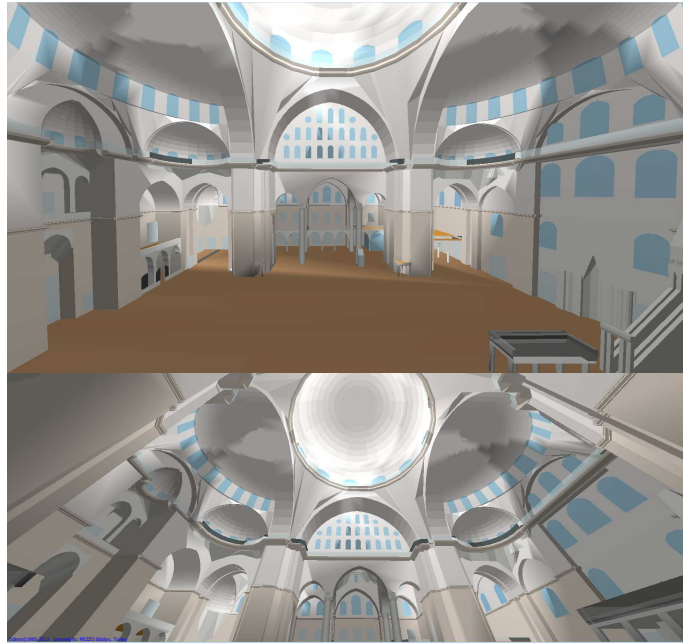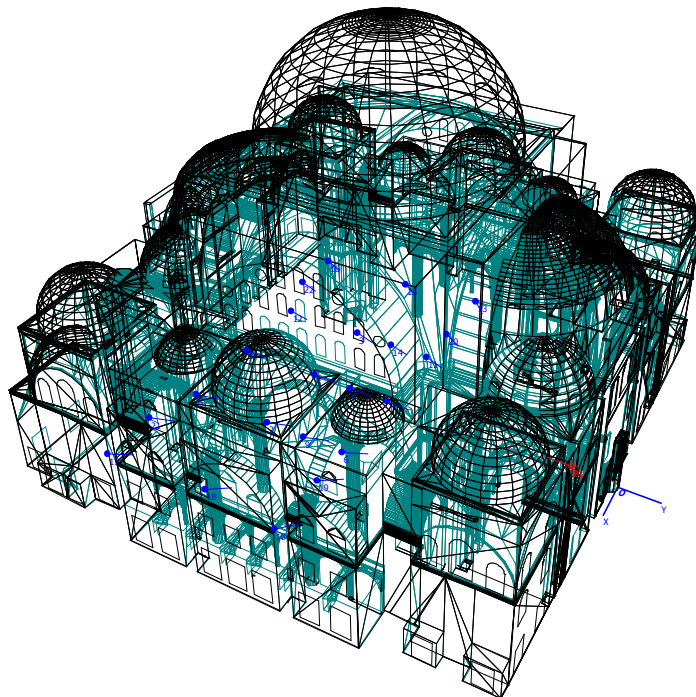
Fig. 2 - 3D-OpenGl view of modeled Mosque

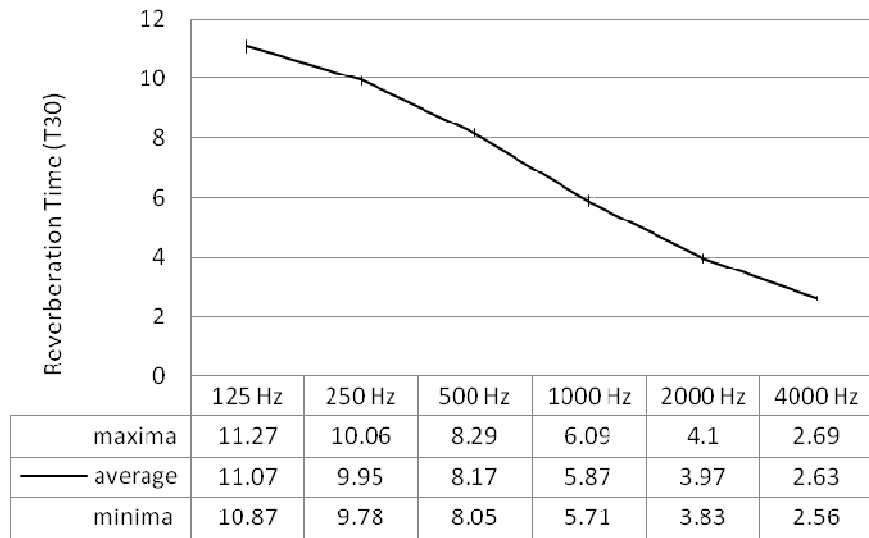*Fig. 3 - ODEON acoustical model with source (red) and receiver (blue) positions*

Fig. 4 - Reverberation time (T30) averages of ODEON simulated data for 22 receiver points over octave bands

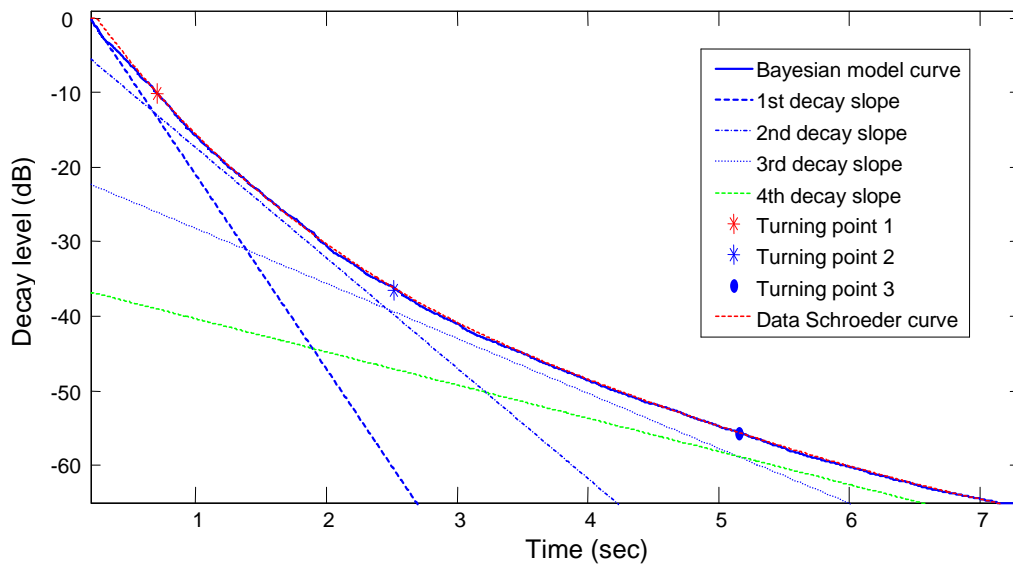|	125 Hz	250 Hz	500 Hz	1000 Hz	2000 Hz	4000 Hz
maxima	11.27	10.06	8.29	6.09	4.1	2.69
average	11.07	9.95	8.17	5.87	3.97	2.63
minima	10.87	9.78	8.05	5.71	3.83	2.56

Fig. 5 - Comparison between Schroeder decay curve and the decay model curve using a quadraple-slope model derived from the room impulse response for R_{10}, bandpass-filtered at 4 kHz; four decomposed decay slope lines and three turning points