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SCHROEDER DECAY DECOMPOSITION FOR SOUND-ENERGY DECAY ANALYSIS IN ACOUSTIC COUPLED-VOLUME SYSTEMS

Ning XIANG, Tomislav JASA, and Zuhre SU
Graduate Program in Architectural Acoustics
School of Architecture, Rensselaer Polytechnic Institute, Troy, NY 12180, USA
E-mail: xiangn@rpi.edu

ABSTRACT

Measurements of multiple decay times in acoustically coupled spaces are of practical significance when studying and designing acoustics of the coupled spaces. Previous work [Xiang JASA **98** (1995) 2112-21, Xiang & Goggans JASA **110** (2001) 1415-24] has applied a parametric model of Schroeder decay functions for sound energy decay analysis. This paper discusses recent development of efficient decompositions of Schroeder decay model terms within Bayesian framework for decay parameter estimations. As relevant decay parameters, in addition to decay times, this paper introduces numerical methods for simultaneous estimations of level differences, turning points, decay time-related derivations. This work shows that Bayesian probabilistic inference can be used as powerful tools for sound energy decay analysis in both coupled spaces and single-space halls. Implemented routines of Bayesian decompositions will be demonstrated over different frequency ranges using experimentally measured room impulse responses in concert halls, churches, real-sized and scale-modeling coupled-volume systems.

KEYWORDS: Room-Acoustics, coupled spaces, Bayesian parameter estimation, Schroeder curve decomposition.

1. INTRODUCTION

With increasing research activities [1-4] in acoustically coupled spaces, analysis of sound energy decay characteristics becomes a highly required task which is to evaluate different decay times and related parameters from single-, or double-slope decay characteristics of Schroeder decay functions [5] using measured or simulated room impulse responses. Traditionally however, identification of double- or multiple- sloped decay in room impulse response measurement has been considered very challenging. Bayesian inference has been applied in coping with the demanding tasks in estimating multiple decay times from Schroeder decay functions [6-8]. In addition to Bayesian decay time estimation, this paper also introduces definition of level differences and turning points once double-slope decay has been identified. Using experimentally measured data in concert halls, churches, real-sized and scale-modeling coupled-volume systems, this paper demonstrates functioning tools to cope with demanding tasks in current research and practice.

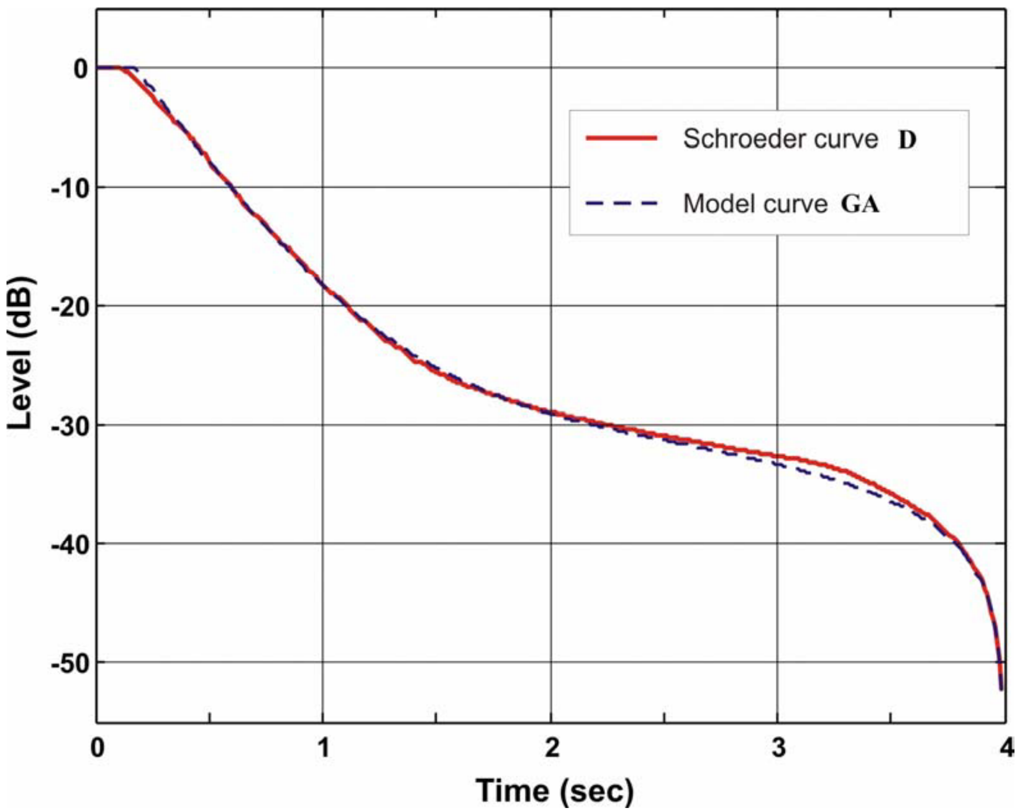

***Figure 1**. Schroeder decay function measured in Sant. Patrick Church, Watervliet, NY, along with the decay model function.*

2. BAYESIAN FORMULATION

2.1 Schroeder decay models

This section begins with Schroeder decay function data $\mathbf{D} = [d_1, d_2, \ldots, d_K]^T$. Derived from the nature of Schroeder's integration, a Schroeder decay function model has been established

$$\mathbf{D} = \mathbf{GA} + \mathbf{e} \tag{1}$$

which approximates the data **D** with an error vector e. **A** is a column vector of *m* coefficients, termed *linear parameter vector*. **G** is a matrix of $K \times m$. *j*th column of **G** is given by

$$G_{kj}(T_j, t_k) = \begin{cases} \exp(-13.8 \cdot t_k / T_j) & \mathit{for}\ j = 1,2,\ldots,m-1 \\ t_K - t_k & \mathit{for}\ j = m \end{cases}, \quad (2)$$

where T_j is jth decay time to be determined for $1 \leq j < m$, $T_m = \infty$. t_K represents the upper limit of Schroeder's integration. $0 \leq k < K-1$. Recent works [6-8] have experimentally proven the validity of this model. Figure 1 shows one example of Schroeder decay function evaluated from real hall measurements in Sant. Patrick Church, Watervliet, NY and its model function $\mathbf{GA}$ with properly estimated model parameters.

We begin with eigen-decomposition of $m \times m$ square matrix:

$$\mathbf{G}^\mathbf{T}\mathbf{G} = \mathbf{E}\mathbf{\Lambda}^\mathbf{T}\mathbf{E}^\mathbf{T}, \quad (3)$$

where $(\cdot)^T$ represents matrix transpose, $\mathbf{\Lambda}$ is a diagonal matrix containing m eigenvalues of eigenvector **E**. The eigen-decomposition facilitates converting **G** into an orthonornalized model

$$\mathbf{Q} = \mathbf{G}\mathbf{E}\mathbf{\Delta}^{-1}, \quad (4)$$

with

$$\mathbf{\Delta}^\mathbf{T}\mathbf{\Delta} = \mathbf{\Lambda}. \quad (5)$$

In similar fashion, the linear parameter vector **A** can be converted into the orthonormalized one or vice verse:

$$\boldsymbol{\alpha} = \mathbf{\Delta}\mathbf{E}\mathbf{A}; \ \ \mathbf{A} = \mathbf{E}\mathbf{\Delta}^{-1}\boldsymbol{\alpha}, \quad (6)$$

such that the error function **e** can be equivalently expressed in terms of Schroeder decay function **D** and the orthonormalized model $\mathbf{Q}\boldsymbol{\alpha}$:

$$\mathbf{e} = \mathbf{D} - \mathbf{Q}\boldsymbol{\alpha}. \quad (7)$$

2.2 Bayesian decay parameter estimation

Bayesian theory formulates the *posterior probability distribution function* (PPDF) through the prior probability distribution and likelihood function via Bays' theorem:

$$p(\boldsymbol{\alpha}, \mathbf{T} \mid \mathbf{D}, I) = \frac{p(\boldsymbol{\alpha}, \mathbf{T}, I)\, p(\mathbf{D} \mid \boldsymbol{\alpha}, \mathbf{T}, I)}{p(\mathbf{D}, I)}, \quad (8)$$

where $p(\mathbf{D}, I)$ acts in the context of decay time estimation as a normalization constant. **T** is a vector matrix of m coefficients, termed *nonlinear parameter vector*. $p(\boldsymbol{\alpha}, \mathbf{T}, I)$ is the prior distribution function of $\boldsymbol{\alpha}$ and T. Background information I includes that the Schroeder decay model in eq. (2) through eq. (1) describes the data **D** reasonably well so that all errors **e** are bounded by a finite value. Given the finite error and a reasonable model as only available information, application of the principle of the maximum entropy assigns a Gaussian distribution to the likelihood function $p(\mathbf{D} \mid \boldsymbol{\alpha}, \mathbf{T}, I)$ and an independence to errors

e_i from each other, so that

$$p(\boldsymbol{\alpha},\mathbf{T}\,|\,\mathbf{D},\sigma,I) \propto p(\boldsymbol{\alpha},\mathbf{T},I)(2\pi\sigma)^{-K} \exp\left[-\frac{(\mathbf{D}-\mathbf{Q}\boldsymbol{\alpha})^{\mathrm{T}}(\mathbf{D}-\mathbf{Q}\boldsymbol{\alpha})}{2\sigma^2}\right]. \tag{9}$$

The posterior function $p(\boldsymbol{\alpha},\mathbf{T}\,|\,\mathbf{D},\sigma,I)$ implies that the error variance σ^2 at this stage is still unknown. The marginalization over $\boldsymbol{\alpha}$ with a uniform prior, and over σ by assigning Jeffreys' prior leads to an analytically tractable PPDF in form of the student-T distribution:

$$p(\mathbf{T}\,|\,\mathbf{D},I) \propto \left[\mathbf{D}^{\mathrm{T}}\mathbf{D}-\mathbf{q}^{\mathrm{T}}\mathbf{q}\right]^{(m-K)/2} \tag{10}$$

with

$$\mathbf{q} = \mathbf{Q}^{\mathrm{T}}\mathbf{D}. \tag{11}$$

The marginalization results in the student-T PPDF over only the decay time space, being independent on linear parameter vector **A**. So the decay time estimation can be carried out in dramatically reduced dimensionality. Once the decay times $\mathbf{T}$ are estimated, an expected linear parameter vector $\hat{\mathbf{A}}$ can be determined [6].

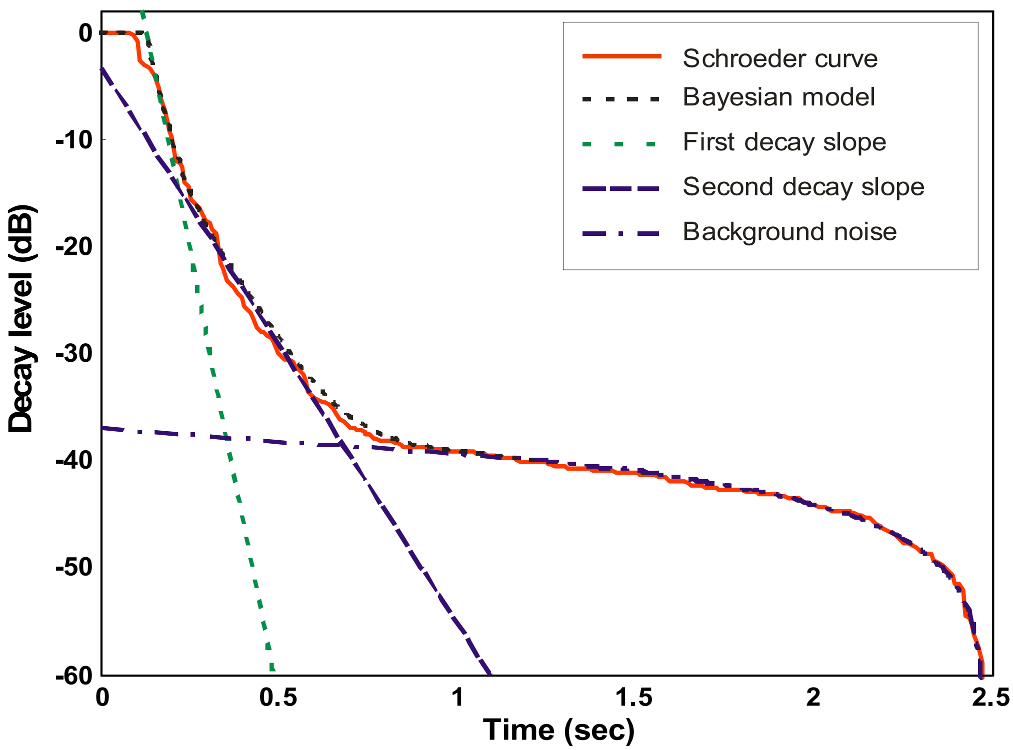

Figure 2. *Comparison between measured decay function (Schroeder curve) in scale model coupled spaces and the decay model function. The model function is decomposed into 3 terms: 1st slope, 2nd slope and the term associated with background noise.*

3. SCHROEDER FUNCTION DECOMPOSITION

Figure 2 shows a comparison between a measured Schroeder decay function (Schroeder curve) in scale model coupled spaces and the decay function model given in Eq. (2). In this example a two-slope decay function model is used within the Bayesian decay time estimation. Three model terms (1st slope, 2nd slope and the term associated with the background noise) are also shown in Fig. 2.

Previous work [6] pointed out, architectural acousticians are primarily interested in the relative difference ΔA between the linear parameters (A_1, A_2), in addition to both decay times (T_1, T_2). Figure 2 shows that the Schroeder decay function decays only after the time point of direct sound. The plateaus at 0 dB at the beginning of the normalized decay curve reflects the fact that prior to the direct sound the sound energy does not decay at all. In practical measurements the direct sound varies significantly depending on the source-receiver positions. In addition, the data analysis should generally begin at the decay level of -5 dB (as for reverberation time analysis). The Bayesian decay decomposition within this work is carried out also in a similar fashion: all the data are taken from -5dB till the end of the data record, regardless of single- or double-slope cases.

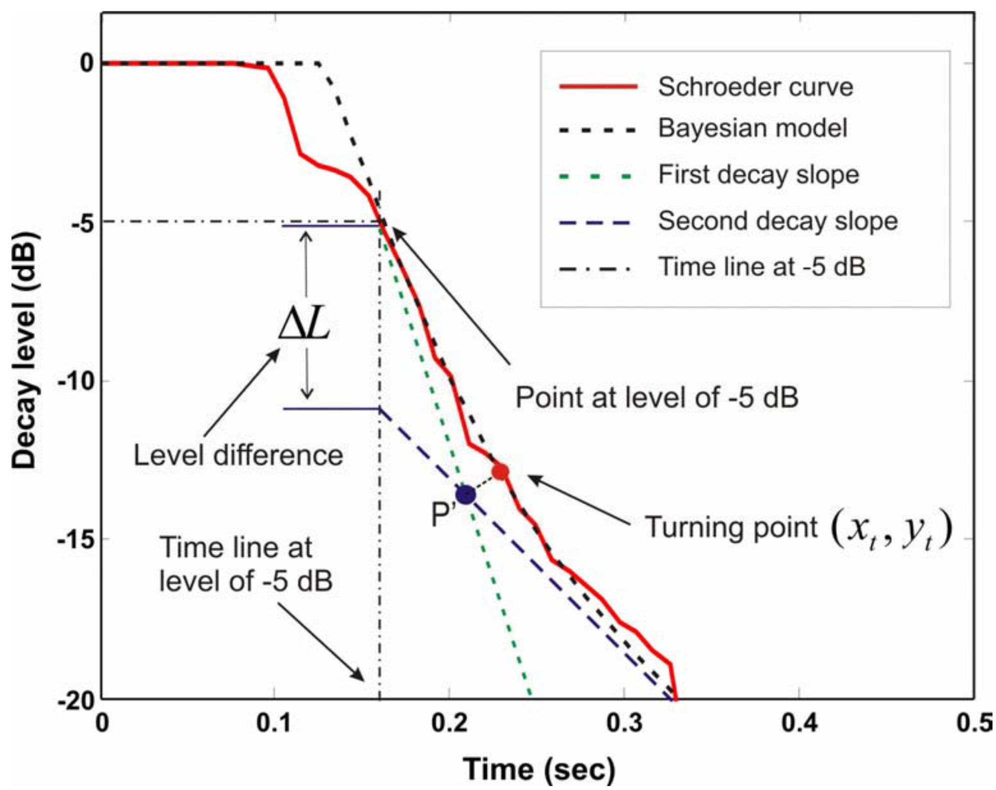

Figure 3. *Definition of level difference ΔL and turning point (x_t, y_t) with help of a zoom into a small portion of Fig. 2.*

3.1 Decay time uncertainty

The decay time estimation within Bayesian framework heavily relies on proper evaluations of PPDF expressed in eq.(10). The mode of the PPDF can be used not only for decay time estimation in terms of maximum *a posterior* (MAP), the shape of the PPDF mode can also be used for uncertainty estimation [8]. To estimate the uncertainties expressed in standard derivations of estimated decay times, sampling methods have been applied based on a Gaussian model, where the co-variance matrix around the PPDF mode are sampled using the importance sampling [8]:

$$\langle C_{ij} \rangle \approx \frac{\sum_{r=1}^{R} (T_{ir} - \langle T_i \rangle)(T_{jr} - \langle T_j \rangle) q(\mathbf{T_r})}{\sum_{r=1}^{R} q(\mathbf{T_r})}, \tag{12}$$

with

$$\langle \mathbf{T} \rangle \approx \frac{\sum_{r=1}^{R} \mathbf{T_r} q(\mathbf{T_r})}{\sum_{r=1}^{R} q(\mathbf{T_r})}, \tag{13}$$

and

$$q(\mathbf{T_r}) \approx \frac{p(\mathbf{T_r} \mid D, I)}{g(\mathbf{T_r})}, \tag{14}$$

where $p(\mathbf{T_r} \mid D, I)$ is given in eq.(10) and $g(\mathbf{T_r})$ is Gaussian random process. Besides the importance sampling, a recent effort is to develop more effective sampling methods such as slice sampling or nested sampling [10].

3.2 Level difference

To avoid arbitrariness, a definition of the level difference is introduced. Upon Schroeder decay function decomposition in case of double-slope decay, two straight lines corresponding to two decay slopes in the logarithmic presentation can be determined. Due to arbitrariness of the direct sound, one needs to determine the crossing points of the two decay lines with the time line of -5dB in the logarithmic scale as show in Fig. 2:

$$y_1 = a_1 + b_1 \cdot t_k, \tag{15}$$

$$y_2 = a_2 + b_2 \cdot t_k, \tag{16}$$

with

$$a_j = 10 \cdot \lg A_j, \tag{17}$$

$$b_j = -10 \cdot \lg e \cdot (13.8 / T_j), \tag{18}$$

where T_j s and A_j s are estimated in terms of Bayesian analysis of Schroeder decay decomposition [6,8]. The time line of – 5dB is determined along the measured decay function.

The level difference ΔL (in dB) is then defined as the logarithmic of the ratio between the two crossing points, or the difference of the two crossing points in the logarithmic scale (as shown in Fig. 2):

$$\Delta L = (y_1 - y_2)|_{t_k \ at -5dB} \qquad \text{(dB)} \tag{19}$$

3.3 Turning point

Some authors [9] also studied the location of a 'turning point' at which the early rapid decay intersects with the slower decay. Extensive investigations on a large number of measured results show that this turning point will not easily be recognized by a visual inspection. An algorithmic approach is needed in practice. Upon Schroeder decay function decomposition in case of double-slope decay, two straight lines corresponding to two decay slopes will in general not co-indent with the data (Schroeder curve) rather lower than the data, the crossing point $P'(x_0, y_0)$ of two straight decay lines to each other is given by:

$$x_0 = (a_2 - a_1)/(b_1 - b_2)\,. \tag{20}$$
$$y_0 = (a_2 b_1 - a_1 b_2)/(b_1 - b_2)\,, \tag{21}$$

which is often way off the Schroeder decay curve and the model decay curve. The turning point $P_t(x_t, y_t)$ is defined to be a point on the decay model curve, to which the crossing point (P') has the minimum distance:

$$\sqrt{(x_t - x_0)^2 - (y_t - y_0)^2} \rightarrow \min\,. \tag{22}$$

Two decay times or decay ratio (T_2/T_1), along with the level difference (ΔL in dB), the co-ordinate of the turning point (x_t, y_t) will be eventually used for further studies of acoustics in coupled spaces.

Table I. Measured results over octave bands, analyzed using Bayesian approaches. In addition to the decay times, their standard derivations, the level difference, the turning point co-ordinates are expressed in time (T_t) and level(L_t).

Band (Hz)	**T_1 (sec)**	**Std_1 (sec)**	**T_2 (sec)**	**Std_2 (sec)**	**ΔL (dB)**	**Turning point**	
						T_t(sec)	**L_t(dB)**
125	0.70	5.51E-3	2.80	3.62E-2	2.51	0.28	-10.42
250	0.69	2.81E-3	2.40	2.38E-2	3.65	0.28	-11.43
500	0.99	3.06E-3	2.74	4.42E-2	7.22	0.38	-16.93
1000	0.87	1.88E-3	2.57	5.97E-2	9.34	0.41	-19.44

4 EXPERIMENTAL RESUTLS

Table I shows some experimental results measured in real coupled spaces, the Student Union of the University of Mississippi, USA. Table I lists the analyzed results of one room impulse response near the coupled area. Over octave bands between 125 Hz and 1 kHz, the two decay times, their standard derivations, the level differences and the turning point co-ordinates are listed. At this measurement position, only one decay slope has been found higher than 1 kHz octave bands. This confirms experimentally that the double-slope phenomena are frequency-dependent [2].

5 SUMMARY

This work describes the Bayesian analysis for Schroeder decay function decomposition. The work is of significance when studying acoustics in coupled spaces. Recent research activities show that the modeling effort, the objective and subjective investigations require a quantitative description of systematic changes in acoustically coupled spaces. One of important tasks is to quantify sound energy decay characteristics. Schroeder decay functions in coupled spaces are decomposed using a model-based Bayesian approach. This work demonstrates that model-based Schroeder decay decomposition provides decay times, level differences of two decay processes and the turning point along the decay function. Along with decay time-related derivations and inter-dependences evaluated within Bayesian framework, these relevant decay parameters will be used in the near future for further study of acoustics in coupled spaces.

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