inter.noise 2012
august 19-22
new york city, usa
Quieting the World's Cities

Incorporation of double leaf micro perforated panels into Schroeder diffusers as a space absorber

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The Schroeder Diffusers are developed for enhanced scattering while sustaining some degree of absorption. Previous researches have demonstrated that the amount of absorption can be improved at low frequencies by modifying the diffuser structure through use of perforated plates in some of the wells. This study aims to observe absorption characteristics of Schroeder Diffusers using double leaf micro perforated panels without a rigid backing. This type of panel system enables the diffuser to function as a space diffuser and improves absorption even at very low frequencies compared to previous studies. The theory of enhanced absorption as well as the transmission loss through non-rigid panel backing is presented in the study. The results show that use of double layer micro perforated panels without a rigid backing improves absorption characteristics of Schroeder diffusers particularly at very low frequencies.

1 INTRODUCTION

Profiled diffusers, namely Schroeder Diffusers were first introduced by Schroeder. These diffusers are periodic structures that are constructed to scatter the incident sound energy rather specularly reflect it. The structure of the diffuser is simply composed of wells having different depths separated with rigid walls. The depth sequence of the wells is arranged according to pseudo-stochastic number sequence.

The theory behind the diffuser is simply based on the phase difference between the reflected sound waves which are incident upon each well with different well depths. The time required for the sound wave to travel inside each well is different, yielding reflected sound waves with different phases. This phase difference grants scattered reflection, generating diffuse field at

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the far field. The depth sequence of the diffuser can be determined by either quadratic residue sequence or primary residue sequence. Generally; quadratic residue sequence is employed for Schroeder diffusers, known as quadratic residue diffusers (QRD) in practical applications.

Quadratic residue diffusers are utilized to disperse the incident sound to create diffuse sound field in the far field; however, there have been several researches on the absorption characteristics of QRDs to observe the amount of absorption that can be harnessed through it. K. Fujiwara and T. Miyajima's work[1] at the end of the last century put some effort to observe the amount of absorption of QRDs experimentally. Surprisingly, they observed high degree of absorption, later it was turned out that the amount of absorption was a result of poor construction of the QRD. In the same decade, H. Kuttruff developed a mathematical theory[2] to model the absorption mechanism in QRDs which was based on uniform sound field at the surface of the diffuser by taking average surface admittance of the wells. A more convenient model was developed by P. F. Mechel[3] who studied scattering and absorption mechanism of Schroeder Diffuser by investigating the mutual interaction of the wells with Fourier analysis. Mechel constructed a Fourier decomposition model to consider the coupling between differently tuned wells more rigorously. Mechel's model was very well developed to understand the behavior of the sound field at the entrance of the wells.

More recent studies were carried out at 2000s using Fourier decomposition method that Mechel introduced. T. Wu, T. J. Cox and Y. W. Lam's work[4] focused on mainly the amount of absorption that a QRD provides and based on a mathematical model to improve absorption characteristic by modifying the structure of the diffuser. Thermal and viscous boundary losses were introduced in the study by narrowing QRD wells to increase the amount of absorption in which the diffuser is transformed to a profiled absorber rather than a profiled scatterer. Following study of T. Wu, T. J. Cox and Y. W. Lam[5] suggested to implement perforated panels at the end of diffuser wells in order to further increase the amount of absorption by inserting additional acoustic mass into the structure. Both works studied the effect of adding a resistive layer, in the form of a wire mesh in front of the diffuser, on the absorption mechanism of the QRD. Results obtained from the theoretical model are compared with the experimental results which comply the reliability of the mathematical theory.

Similar approach regarding the addition of perforated panels is tried by Hunecke[6], by using micro-perforated panels, to obtain both diffusion and absorption characteristics of Schroeder diffusers. The study showed that effect of micro-size perforation enhances absorption in low frequency bands while causing a reduction in scattered sound energy due to the significant amount of absorption in that particular frequency bands. All developed models so far are based on the definition of the absorption mechanism of quadratic residue diffusers either mathematically or experimentally where the QRD is employed with a rigid backing. This particular study focuses on the mathematical model of quadratic residue diffusers utilized with a non rigid backing facing to free field when micro-perforated panels are involved in the structure.

Micro-perforated panel theory is first introduced by Maa[7] in which the acoustic impedance of sub-millimeter size apertures and other design parameters are defined. There are several ways to implement micro-perforated panels (MPP) to absorber devices. Most common uses of MPPs are single and double leaf panel absorbers. Single leaf MMP with a rigid backing introduces a resonance frequency acting as a Helmholtz resonator. Inserting additional MPPs, placed with an air gap, results in a number of distinct resonance frequencies yielding a wide band resonant absorber. To increase the effectiveness of the panel absorber further, K. Sagakami, M. Morimoto and W. Koike[8] studied on a double-leaf MPP with an air gap between and without an air back cavity to ignore the effect of rigid backing. Avoiding rigid backing provides effective absorption at very low frequencies where conventional types suffer.

In this study, a different use of 1-D quadratic residue diffusers using micro-perforated panels without rigid backing is proposed. Since a rigid backing is not required, QRD is not necessarily mounted on a rigid wall or ceiling, it can be used as a space diffuser where both plane surfaces are exposed to free field. The theory behind the reflection mechanism, as well as the transmission mechanism, is concluded to demonstrate the absorption behavior of the QRD.

2 THEORETICAL METHOD

In order to predict the reflection at the surface of the diffuser, Fourier decomposition method of Mechel is used. Mechel's analysis more accurately considers the coupling between the wells compared with the average admittance method; hence the mutual interactions between each well can be examined in a correct manner. This analysis can only be applied to periodic structures. Due to the nature of the QRD, the depth sequence is going to be periodic when infinitely many QRDs are mounted along one direction. Since the structure is periodic, the pressure field in front of the diffuser is also periodic and can be expanded by Fourier decomposition.

Figure 1 shows a schematic view of a 1-D QRD with the direction of the coordinate axes and the parameters that are going to found the theory. A very well known formula for the depth sequence of the QRD by Schroeder is described as;

$$l_n = \frac{c \bmod(n^2, N)}{N 2 f_r}, \tag{1}$$

where l_n is the depth of the nth well, c is the speed of sound in the medium

, f_r is the design frequency and N is the prime number that constructs the quadratic residue sequence.

The following absorption analysis is going to be performed on 1-D QRD where the horizontal panels are micro-perforated panels with an air back cavity between and the rear panel is exposed to free field. The depth of the cavity between each double leaf micro-perforated panel system, d_n, is determined as l - l_n, where l is the maximum depth determined by the design wavelength of the QRD.

2 PREDICTION OF ABSORPTION

2.1 The Impedance of the Micro-perforated Panel

Micro-perforated panels have been effective panel absorbers for some years due to their potential to absorb sound energy at low frequencies. They are simply considered as a lattice of short tubes with sub-millimeter sizes where the distance between each tube is small compared to the wavelength of incident sound wave. Since the resonance frequencies of both the panel and the apertures are in the same range, their simultaneous effect should be considered. This interaction is described as a parallel connection of impedances of both apertures and the panel. Hence; the specific acoustic impedance of a single layer micro-perforated panel can be obtained by using electric circuit analogy as shown in Figure 2. ρc term at the end of the circuit is for the free field impedance at the back of the diffuser.

The normalized specific acoustic impedance of the panel and the apertures can be formulated as;

$$Z_M^k = \frac{R_M^k + jM_M^k}{\rho c}, Z_L^k = \frac{R_L^k + jM_L^k}{\rho c}. \quad (2)$$

Each parameter in Eqn. (2) can be found in the study of J. Kang and H. V. Fuchs[9], and k is for the number of micro-perforated panels used. Once the impedances of both apertures and the panel are obtained, the normalized specific acoustic impedance of the micro-perforated panel can be calculated by the electric-circuit analogy where both impedances are connected in parallel;

$$Z_m = Z_T + \left[\left(\frac{1}{Z_T + 1}\right) + \frac{1}{Z_{d_n}}\right]^{-1}, \quad (3)$$

where $Z_{dn} = -j\cot(kd_n)$ and Z_T is the total normalized acoustic impedance of single MPP layer. For simplicity, micro-perforated panel parameters are taken as the same.

2.2 Prediction of Reflection by QRD

Prediction of reflected energy from the front surface of the QRD is handled by the method explained by Mechel[11]. The sound field in front of the diffuser is the sum of incident plane wave and the scattered sound field determined by the Fourier expansion.

$$P_e(x,z) = P_e \exp(-ik_x x + ik_z z), \quad (4)$$

$$P_s(x,z) = \sum_{n=-\infty}^{\infty} A_n \exp(-i\beta_n x - i\gamma_n z). \quad (5)$$

Since QRD is periodic, the wave number in x direction is also periodic and described as;

$$\beta_n = k_x + n\frac{2\pi}{T}. \quad (6)$$

From dispersion relation wave number in z direction can be obtained as;

$$\gamma_n = -jk\sqrt{(Sin\theta + n\frac{\lambda}{T})^2 - 1}. \quad (7)$$

The boundary condition at the surface of the QRD can be represented by the relation between the sound pressure field and the normal velocity field using specific normalized acoustic admittance, *G(x)*; which can be obtained by a transfer matrix method as the inverse of the specific acoustic impedance;

$$z(x) = \frac{\rho c}{G(x)} = \frac{\rho c Z_m \coth(jkl_n) + (\rho c)^2}{Z_m + \rho c \coth(jkl_n)}. \quad (8)$$

The relation between P_e and P_s is derived from the boundary condition at the front surface of the QRD as follows;

$$\cos\theta_e P_e - \sum_{n=-\infty}^{\infty} \frac{\gamma_n}{k} A_n \exp(-jn\frac{2\pi}{T}x) = G(x)\left[P_e + \sum_{n=-\infty}^{\infty} A_n \exp(-jn\frac{2\pi}{T}x)\right]. \qquad (9)$$

Since QRD is periodic, *G(x)* can be represented by Fourier expansion;

$$G(x) = \sum_{n=-\infty}^{\infty} g_n \exp\left(-jn\frac{2\pi}{T}x\right),$$

$$g_n = \frac{1}{T}\int_0^T G(x)\exp\left(jn\frac{2\pi}{T}x\right)dx. \qquad (10)$$

After inserting Eqn. (10) into the Eqn. (9), multiplying with $\exp\left(jm\frac{2\pi}{T}x\right)$ and integrating over the period *T*, the following equation is obtained.

$$\sum_{n=-\infty}^{\infty} A_n\left[g_{m-n} + \delta_{m,n}(\frac{\gamma_n}{k})\right] = P_e(\delta_{m,0}\cos\theta_e - g_m),$$

$$m = -\infty............\infty. \qquad (11)$$

The magnitudes of the reflected energy, A_n, is the solution of Eqn. (11) where infinitely many linear equations should be solved simultaneously. However; the solution converges at the index limits at $n,m = \pm 2N$, hence, further calculations are not necessary. Then, the reflection coefficient as a solution of Eqn. (11) is calculated as;

$$r(\theta) = \left|\frac{A_0}{P_e}\right|^2 + \frac{1}{\cos\theta_e}\sum_{n_s\neq 0}\left|\frac{A_{n_s}}{P_e}\right|^2 \sqrt{1-(\sin\theta_e + n_s\frac{\lambda}{T})^2}. \qquad (12)$$

2.3 Prediction of Transmission through QRD

The absorption coefficient of the QRD can be obtained both evaluating the reflection coefficient and transmission coefficient of the device. To predict transmission coefficient, the same procedure can be applied by defining a relation between the incident plane wave and the transmitted sound wave. Since QRD is periodic in x direction, transmitted sound field is also periodic. Wave number in x direction can be defined as in Eqn.(6) due to the periodicity yielding γ_n to be given as in Eqn.(7). In terms of defined wave numbers, the transmitted sound wave becomes as;

$$P_t(x,z) = \sum_{n=-\infty}^{\infty} B_n \exp(-i\beta_n x + i\gamma_n z). \qquad (13)$$

The relation between the incident plane wave and transmitted sound wave is not constructed directly with the specific acoustic admittance. Hence, to define the interrelation, a transfer matrix should be established by the boundary conditions at *z=0*, *z=-l_n* and *z=-l*. When the boundary conditions are gathered in one equation, the interrelation between P_e and P_t is formed as;

$$g_1 P_e + g_2 \sum_{n=-\infty}^{\infty} B_n \exp(-in\frac{2\pi}{T}x) + \sum_{n=-\infty}^{\infty} B_n \frac{\gamma_n}{k} \exp(-in\frac{2\pi}{T}x) = 0 \ . \tag{14}$$

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Equation (14) is expanded with Fourier transform, since the transfer functions g_1 and g_2 are periodic due to the periodicity in the depth sequence. After expansion Eqn.(14) becomes;

$$\sum_{n=-\infty}^{\infty} B_n \left[g_{2_{m-n}} + \delta_{m,n} (\frac{\gamma_n}{k}) \right] = P_e (\delta_{m,0} \cos\theta_e - g_{1_m}) \ , \qquad m = -\infty \ldots\ldots\ldots\ldots \infty \ . \tag{17}$$

The solution of Eqn.(*) when the index limits of the infinitely many linear equations terminate at *n,m =±2N*, are the magnitudes of the propagating harmonics of the transmitted sound wave, B_n. Then the transmission coefficient of the QRD is then given by;

$$t(\theta) = \left| \frac{B_0}{P_e} \right|^2 + \frac{1}{\cos\theta_e} \sum_{n_s \neq 0} \left| \frac{B_{n_s}}{P_e} \right|^2 \sqrt{1 - (\sin\theta_e + n_s \frac{\lambda}{T})^2} \ . \tag{18}$$

3 NUMERICAL RESULTS

The theory explained above is utilized to derive the absorption coefficient of one-dimensional QRD without a rigid backing. To provide numerical results, QRD and micro-perforated panel parameters are assigned as given in Table 1 and Table 2. For simplicity, values that define MPPs are taken as the same. When the given parameters are inserted in–to the mathematical model, the absorption coefficient of the system can be obtained by the well known formula;

$$\alpha(\theta) = 1 - \left[r(\theta) + t(\theta) \right] . \tag{19}$$

Figure 3 illustrates the absorption coefficient of the defined QRD above. The results are given for normal incidence where $\theta_e = 0$. The green line demonstrates the absorption coefficient of the QRD where the transmitted energy loss is not considered. This information gives an insight about how much of the sound energy is absorbed at the surface of the QRD. On the other side; the blue line indicates the absorption coefficient of the QRD where both reflected and the transmitted energies are taken into account. The difference between both lines is the energy loss due to the transmission of sound to the back of the non-rigid panel.

4 CONCLUSIONS

The study mainly focuses on a new approach to employ quadratic residue diffusers as a space diffuser rather than hanging onto ceilings or walls as a conventional approach. Here, it is represented as an introductory work on to predict the transmission loss, as well as the reflected energy, of quadratic residue diffusers hanged over space. This usage of QRD enables low frequency absorption since non-rigid backing panels are used. To introduce additional resonance frequencies, micro-perforated panels are constituted as the lateral panels of the QRD. Through modification of the well width of the diffuser, the dominancy of scattered energy on the absorbed energy can be obtained. Increasing well width results in higher scattered energy ratios. However; this study is interested particularly in the low frequency absorption; hence the well width is kept small.

Smaller well widths introduce boundary losses since the wavelength of the interested frequency range becomes comparable to the viscous and thermal boundary layer thicknesses. Future study on this subject is going to include the effect of boundary losses when boundary layer thicknesses are comparable. Considering the effect of boundary losses is going to give an advantage to the QRD such that the absorbed energy harnessed by the diffuser is going to increase. Additionally, to further increase the frequency range where absorption is dominant, the depth sequence of the diffuser can be modified to evenly distribute resonance frequencies. Several optimization methods can be applied to the theory to enhance absorption in desired frequency ranges.

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Table 1 – QRD parameters used in the mathematical model

QRD Parameters	Values
Prime number *(N)*	7
Design frequency (f_r)	250 Hz
Well width *(b)*	$1x10^{-2}$ m
Fin width *(w)*	$3x10^{-3}$ m
Incident angle (θ_e)	0^o

Table 2 – Micro-perforated panel parameters used in the mathematical model

MPP Parameters	Values
Aperture size (d^1, d^2)	$7x10^{-3}$ m
Perforation ratio (p^1, p^2)	%1
Panel thickness (t^1, t^2)	$1x10^{-3}$ m
Material surface density (m^1, m^2)	$1x10^{-1}$kg/m^2

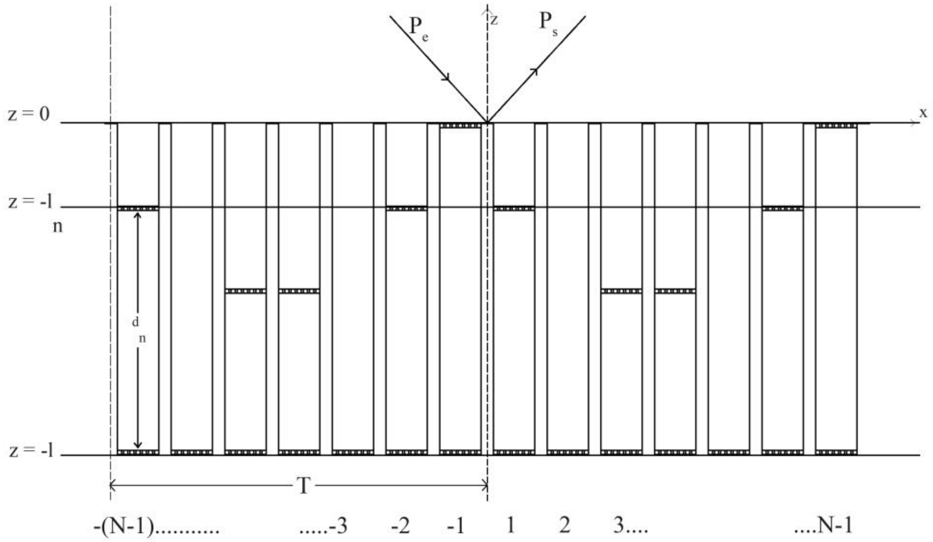

Fig. 1 – One dimensional quadratic residue diffuser for N=7

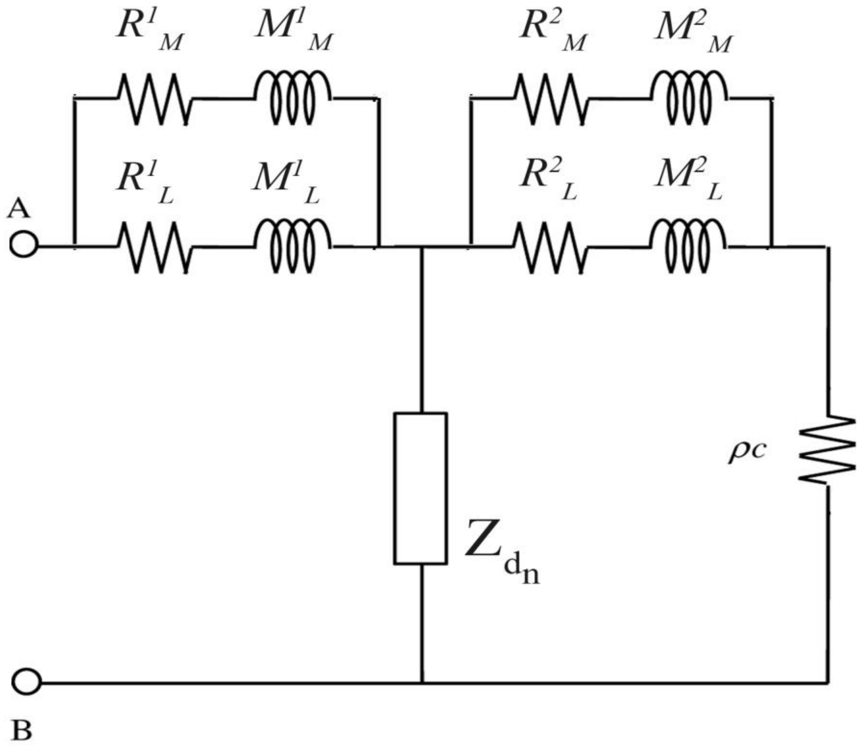

Fig. 2 – Equivalent electric circuit structure of the double layer micro-perforated panel system with an airgap between and non-rigid backing

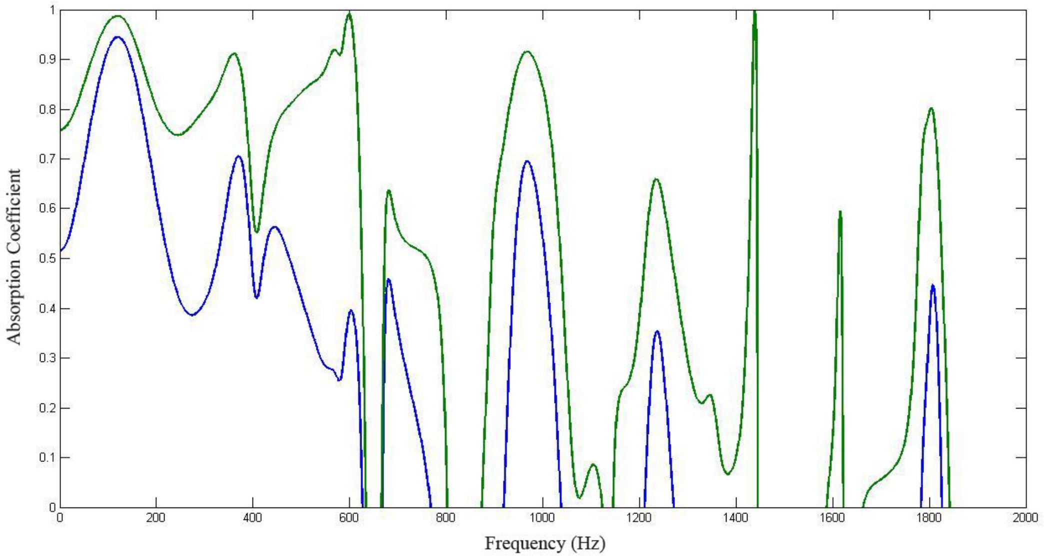

Fig. 3 – Sound absorption coefficient provided with quadratic residue diffuser without a rigid backing under normal incidence (θ_e=0). (Green line: absorption coeff. When only reflected energy is considered, Blue line: absorption coeff. When both reflected and transmited energy are considered.)